

正弦・余弦の加法定理

1 $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

2 $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

加法定理はどのような定理か？

$\sin 75^\circ$

$$= \sin(45^\circ + 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$\cos 75^\circ$

$$= \cos(45^\circ + 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

加法定理の1、2はそれぞれ2つあるが、実は1つ

1 $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

この式の β を $-\beta$ と置き換えると、

$$\sin(\alpha - \beta) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta)$$

$$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

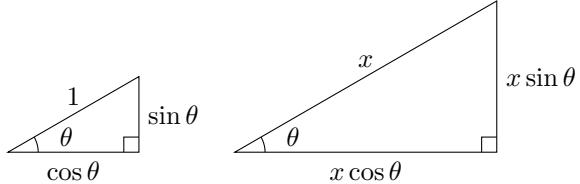
2 $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

この式の β を $-\beta$ と置き換えると、

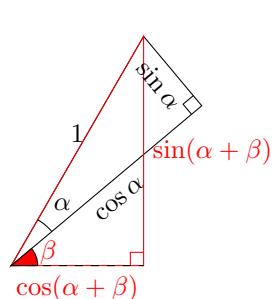
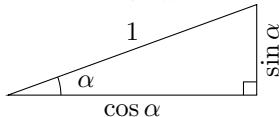
$$\cos(\alpha - \beta) = \cos \alpha \cos(-\beta) - \sin \alpha \sin(-\beta)$$

$$= \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

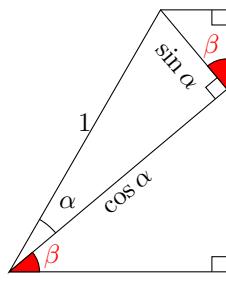
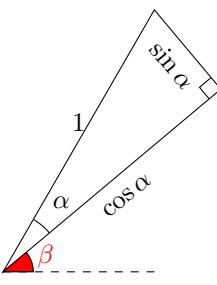
加法定理の証明の準備



加法定理の証明



問1 次の図を利用して、加法定理を導きなさい。



【解答】 $*+*+*+*+*+*+*+*+*$