

次の式を 2 つの三角関数の和か差の形になおせ。

$$\cos 3x \cdot \cos 2x$$



今回の学習目標

三角関数の積和の公式

- 数学 III 微分積分の重要公式

積を和・差になおす公式

$$\boxed{1} \quad \sin \alpha \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$



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$$\boxed{1} \quad \sin \alpha \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

正弦・余弦の加法定理

$$\boxed{1} \quad \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

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次の等式を証明せよ。

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ビデオを止めて問題を解いてみよう

問 1 次の等式を証明せよ。

$$\sin \alpha \sin \beta = -\frac{1}{2} \{ \cos(\alpha + \beta) - \cos(\alpha - \beta) \}$$

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$\cos 3x \cdot \sin x$ を 2 つの三角関数の和か差の形
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$$\boxed{\text{答}} \quad \cos 3x \cdot \sin x = \frac{1}{2} (\sin 4x - \sin 2x)$$



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(1) $\cos 3x \cdot \cos 2x$

(2) $\sin 4x \cdot \sin x$



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ビデオを止めて問題を解いてみよう

問 3 $\sin 75^\circ \cos 15^\circ$ の値を求めよ。

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$$\begin{aligned} & \sin 75^\circ \cos 15^\circ \\ &= \frac{1}{2} \{ \sin(75^\circ + 15^\circ) + \sin(75^\circ - 15^\circ) \} \\ &= \frac{1}{2} \{ \sin 90^\circ + \sin 60^\circ \} \end{aligned}$$

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$$= (\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ) \times \\ (\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ)$$



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$$= \left(\frac{\sqrt{6} + \sqrt{2}}{4} \right)^2 = \frac{6 + 4\sqrt{3} + 2}{16} = \frac{2 + \sqrt{3}}{4}$$



今回の学習目標

三角関数の積和の公式

- 数学 III 微分積分の重要公式