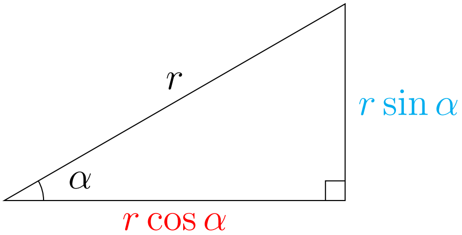
# 2600. 三角関数の合成（一般）

$3\sin\theta + 2\cos\theta$ を $r\sin(\theta+\alpha)$ の形に変形したとき、$r$ の値を求めよ。

$$\begin{array}{cccc} 3 & \sin\theta + & 2 & \cos\theta \\ \downarrow & & \downarrow & \\ r\cos\alpha & & r\sin\alpha & \end{array}$$



# 今回の学習目標

三角関数の合成

- 図を用いて解を導く

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## 三角関数の合成

$A\sin\theta + B\cos\theta$ は、ひとつの波になる。

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$A\sin\theta + B\cos\theta$ は、ひとつの波になる。

$$r\ \cos\alpha\cdot\sin\theta + r\ \sin\alpha\cdot\cos\theta = r\sin(\theta+\alpha)$$

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**例 1**　$3\sin\theta + 2\cos\theta$ を $r\sin(\theta+\alpha)$ の形に変形したとき、$r$ の値を求めよ。

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$$\underset{r\cos\alpha}{\underset{\downarrow}{3}}\sin\theta + \underset{r\sin\alpha}{\underset{\downarrow}{2}}\cos\theta$$

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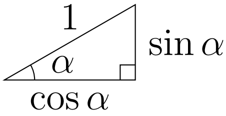
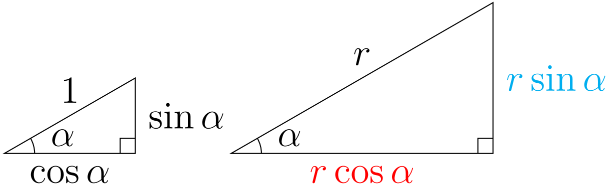


**例 1**　$3\sin\theta + 2\cos\theta$ を $r\sin(\theta+\alpha)$ の形に変形したとき、$r$ の値を求めよ。

$$\underset{\substack{\downarrow\\ r\cos\alpha}}{3}\sin\theta + \underset{\substack{\downarrow\\ r\sin\alpha}}{2}\cos\theta$$



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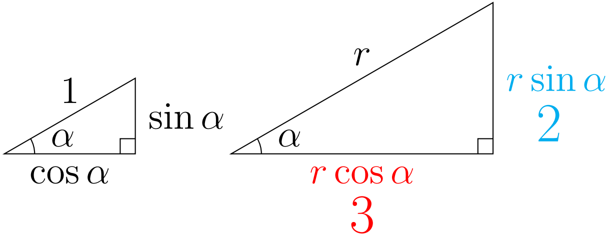


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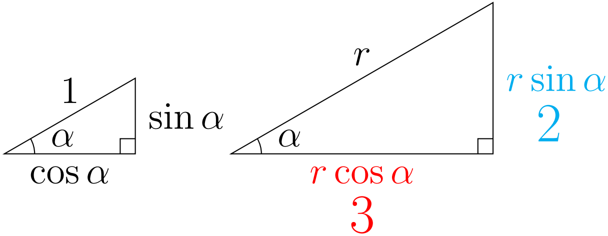
**例 1**　$3\sin\theta + 2\cos\theta$ を $r\sin(\theta+\alpha)$ の形に変形したとき、$r$ の値を求めよ。

$$3\sin\theta + 2\cos\theta$$

$3 \to r\cos\alpha$　　$2 \to r\sin\alpha$




$$r^2 = 3^2 + 2^2 = 13$$

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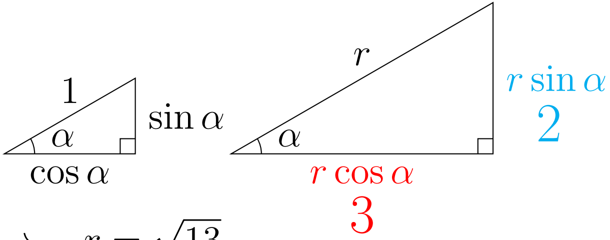
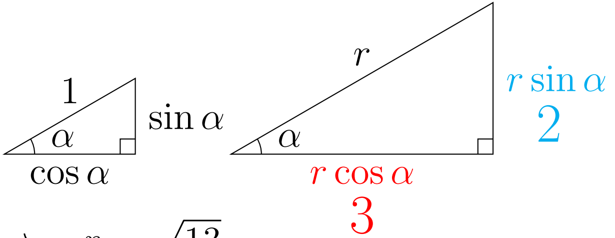



$$r^2 = 3^2 + 2^2 = 13 \quad \rightarrow \quad r = \sqrt{13}$$

**例 1**　$3\sin\theta + 2\cos\theta$ を $r\sin(\theta+\alpha)$ の形に変形したとき、$r$ の値を求めよ。

$$\underset{\displaystyle r\cos\alpha}{\overset{3}{\downarrow}}\ \sin\theta + \underset{\displaystyle r\sin\alpha}{\overset{2}{\downarrow}}\ \cos\theta$$




$$r^2 = 3^2 + 2^2 = 13 \quad \rightarrow \quad r = \sqrt{13}$$

$$\alpha = \tan^{-1}\frac{2}{3} = 33.69^\circ$$

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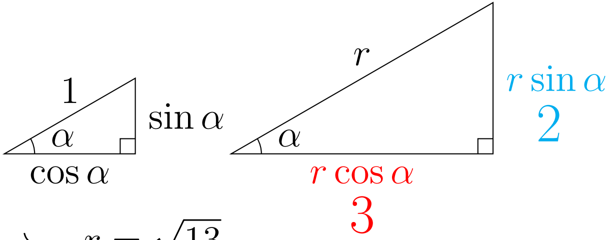


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$$3\sin\theta + 2\cos\theta = \sqrt{13}\cos\alpha \cdot \sin\theta + \sqrt{13}\sin\alpha \cdot \cos\theta$$

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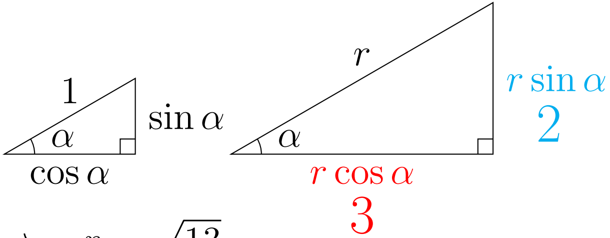


$$r^2 = 3^2 + 2^2 = 13 \quad \to \quad r = \sqrt{13}$$

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$$\begin{aligned} 3\sin\theta + 2\cos\theta &= \sqrt{13}\cos\alpha \cdot \sin\theta + \sqrt{13}\sin\alpha \cdot \cos\theta \\ &= \sqrt{13}\sin(\theta+\alpha) \end{aligned}$$

**例 1**　$3\sin\theta + 2\cos\theta$ を $r\sin(\theta+\alpha)$ の形に変形したとき、$r$ の値を求めよ。

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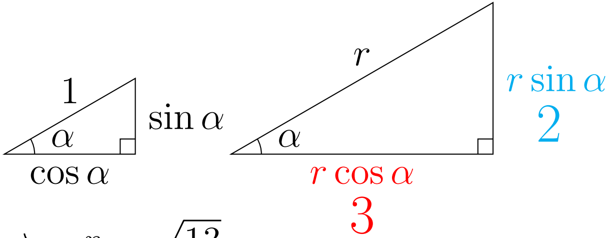


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答　$r = \sqrt{13}$

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# ビデオを止めて問題を解いてみよう

**問 1**　次の式を $r\sin(\theta+\alpha)$ の形に変形したとき、$r$ の値を求めよ。

(1)　$4\sin\theta+3\cos\theta$

(2)　$5\sin\theta-2\cos\theta$

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**問 1**　(1)　$4\sin\theta + 3\cos\theta$

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$$\underset{\downarrow \atop r\cos\alpha}{4}\sin\theta + \underset{\downarrow \atop r\sin\alpha}{3}\cos\theta$$

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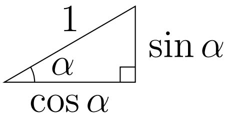
$$\underset{\substack{\downarrow \\ r\cos\alpha}}{4}\,\sin\theta + \underset{\substack{\downarrow \\ r\sin\alpha}}{3}\,\cos\theta$$



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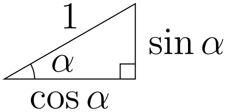
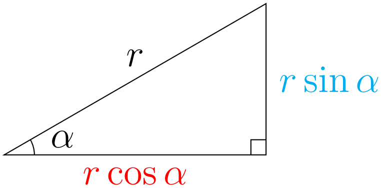
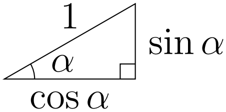
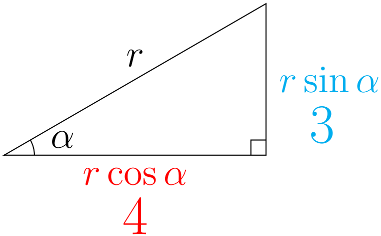


**問 1** (1) $4\sin\theta + 3\cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{4}}\sin\theta + \underset{r\sin\alpha}{\underset{\downarrow}{3}}\cos\theta$$

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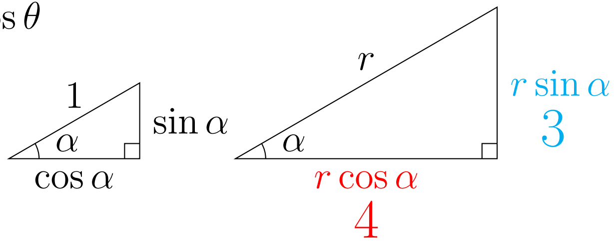
**問 1**　(1)　$4\sin\theta + 3\cos\theta$

$$\underset{\downarrow\atop r\cos\alpha}{4}\ \sin\theta + \underset{\downarrow\atop r\sin\alpha}{3}\ \cos\theta$$




$$r^2 = 4^2 + 3^2 = 25$$

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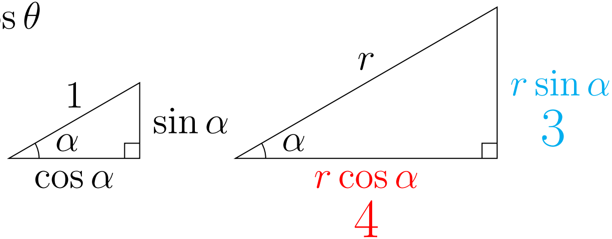


$$r^2 = 4^2 + 3^2 = 25 \quad \rightarrow \quad r = 5$$

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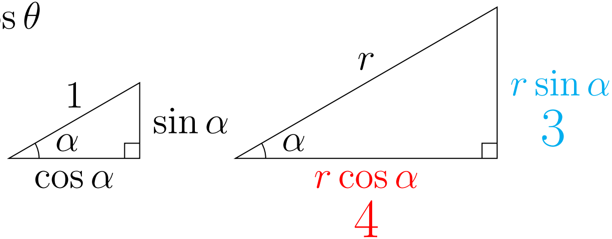


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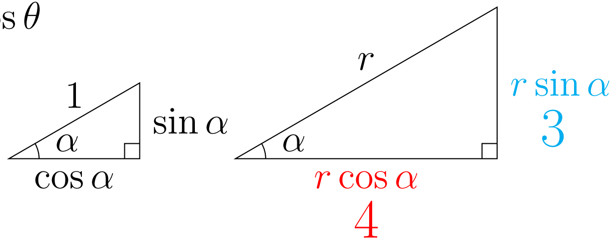


$$r^2 = 4^2 + 3^2 = 25 \quad \to \quad r = 5$$

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$$4\sin\theta + 3\cos\theta = 5\cos\alpha \cdot \sin\theta + 5\sin\alpha \cdot \cos\theta$$

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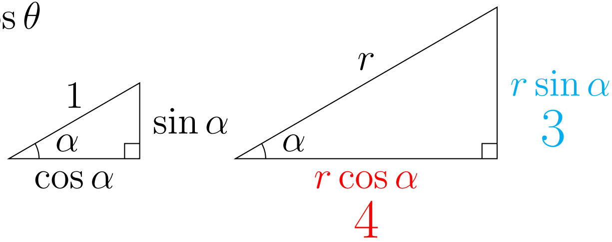


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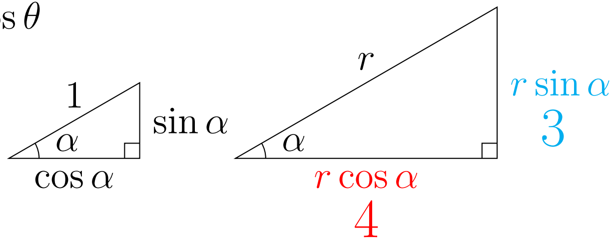


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$$\begin{aligned} 4\sin\theta + 3\cos\theta &= 5\cos\alpha \cdot \sin\theta + 5\sin\alpha \cdot \cos\theta \\ &= 5\sin(\theta + \alpha) \end{aligned}$$

答　$r = 5$

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問 1　(2)　$5\sin\theta - 2\cos\theta$

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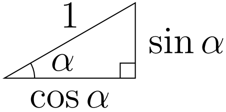
**問 1**　(2)　$5\sin\theta - 2\cos\theta$

$$\underset{\substack{\downarrow \\ r\cos\alpha}}{5}\sin\theta - \underset{\substack{\downarrow \\ r\sin\alpha}}{2}\cos\theta$$

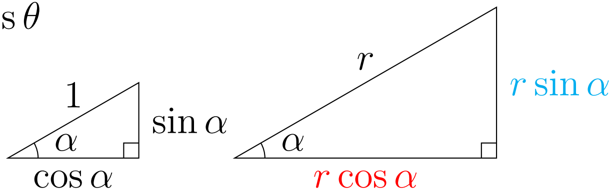
**問 1**　(2)　$5\sin\theta - 2\cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{5}}\sin\theta - \underset{r\sin\alpha}{\underset{\downarrow}{2}}\cos\theta$$



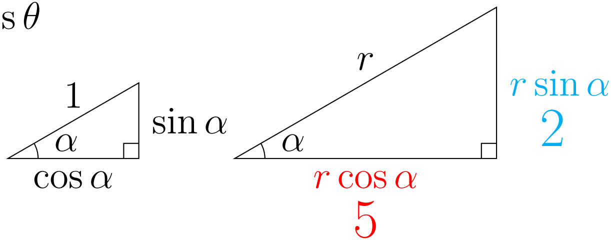
## 問 1　(2)　$5\sin\theta - 2\cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{5}}\sin\theta - \underset{r\sin\alpha}{\underset{\downarrow}{2}}\cos\theta$$



**問 1** (2) $5\sin\theta - 2\cos\theta$

$$\underset{\substack{\downarrow\\ r\cos\alpha}}{5}\sin\theta - \underset{\substack{\downarrow\\ r\sin\alpha}}{2}\cos\theta$$



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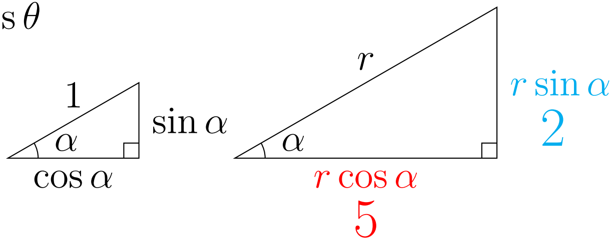
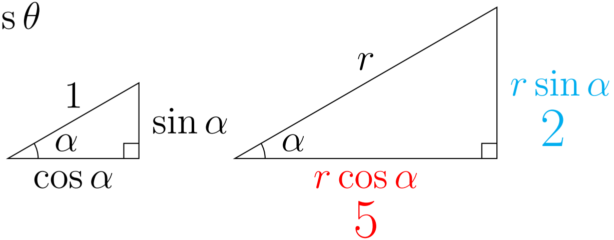
$$\underset{\substack{\downarrow\\ r\cos\alpha}}{5}\,\sin\theta - \underset{\substack{\downarrow\\ r\sin\alpha}}{2}\,\cos\theta$$




$$r^2 = 5^2 + 2^2 = 29$$

**問 1**　(2)　$5\sin\theta - 2\cos\theta$

$$\underset{\downarrow\, r\cos\alpha}{5}\,\sin\theta - \underset{\downarrow\, r\sin\alpha}{2}\,\cos\theta$$




$$r^2 = 5^2 + 2^2 = 29 \quad \rightarrow \quad r = \sqrt{29}$$

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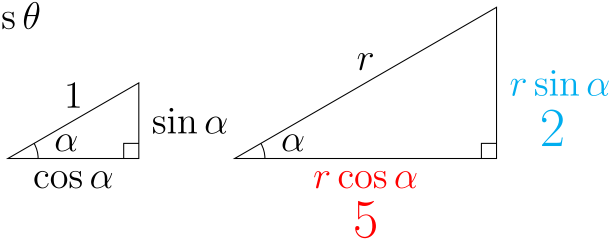
**問 1** (2) $5\sin\theta - 2\cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{5}}\ \sin\theta - \underset{r\sin\alpha}{\underset{\downarrow}{2}}\ \cos\theta$$

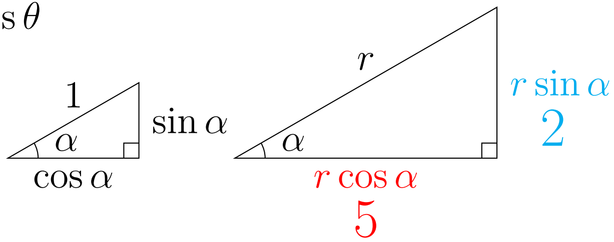



$$r^2 = 5^2 + 2^2 = 29 \quad \rightarrow \quad r = \sqrt{29}$$

$$\alpha = \tan^{-1}\frac{2}{5} = 21.80^\circ$$

## 問 1　(2)　$5\sin\theta - 2\cos\theta$

$$\underset{\color{red}{r\cos\alpha}}{\color{red}{5}}\sin\theta - \underset{\color{cyan}{r\sin\alpha}}{\color{cyan}{2}}\cos\theta$$




$$r^2 = 5^2 + 2^2 = 29 \quad \rightarrow \quad r = \sqrt{29}$$

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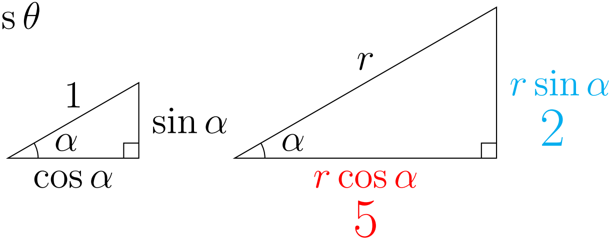
$$5\sin\theta - 2\cos\theta = \color{red}{\sqrt{29}\cos\alpha}\cdot\sin\theta - \color{cyan}{\sqrt{29}\sin\alpha}\cdot\cos\theta$$

## 問 1　(2)　$5\sin\theta - 2\cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{5}}\,\sin\theta - \underset{r\sin\alpha}{\underset{\downarrow}{2}}\,\cos\theta$$



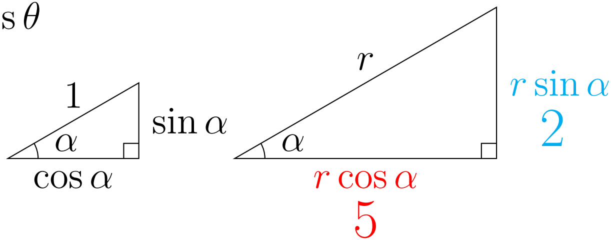

$$r^2 = 5^2 + 2^2 = 29 \quad \rightarrow \quad r = \sqrt{29}$$

$$\alpha = \tan^{-1}\frac{2}{5} = 21.80^\circ$$

$$\begin{aligned} 5\sin\theta - 2\cos\theta &= \sqrt{29}\cos\alpha \cdot \sin\theta - \sqrt{29}\sin\alpha \cdot \cos\theta \\ &= \sqrt{29}\sin(\theta - \alpha) \end{aligned}$$

## 問 1 (2) $5\sin\theta - 2\cos\theta$

$$\underset{\displaystyle r\cos\alpha}{\underset{\downarrow}{5}}\ \sin\theta - \underset{\displaystyle r\sin\alpha}{\underset{\downarrow}{2}}\ \cos\theta$$

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$\boxed{答}$ $r = \sqrt{29}$

# ビデオを止めて問題を解いてみよう

**問 2**　次の式を $r\sin(\theta+\alpha)$ の形に変形せよ。

(1)　$\sqrt{3}\sin\theta+\cos\theta$

(2)　$\sin\theta-\cos\theta$

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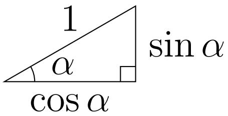
**問 2**　(1)　$\sqrt{3}\sin\theta + \cos\theta$

**問 2**　(1)　$\sqrt{3}\sin\theta + \cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{\sqrt{3}}}\sin\theta + \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$

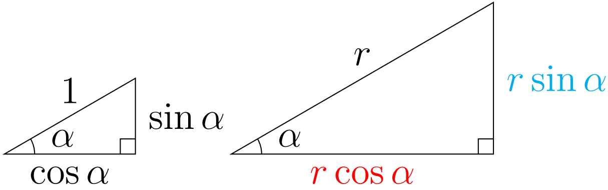
**問 2**　(1)　$\sqrt{3}\sin\theta + \cos\theta$

$$\underset{\displaystyle r\cos\alpha}{\underset{\downarrow}{\color{red}\sqrt{3}}}\sin\theta + \underset{\displaystyle r\sin\alpha}{\underset{\downarrow}{\color{cyan}1}}\cos\theta$$



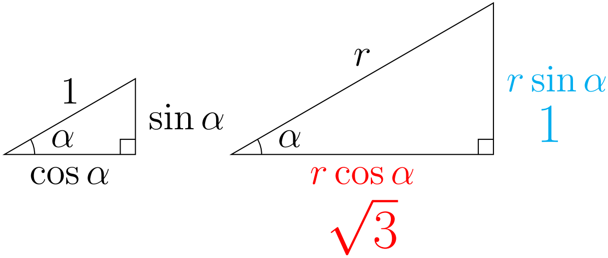
**問 2**　(1)　$\sqrt{3}\sin\theta + \cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{\sqrt{3}}}\sin\theta + \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$



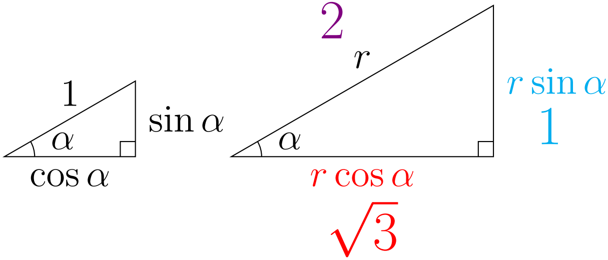
**問 2** (1) $\sqrt{3}\sin\theta + \cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{\sqrt{3}}}\sin\theta + \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$



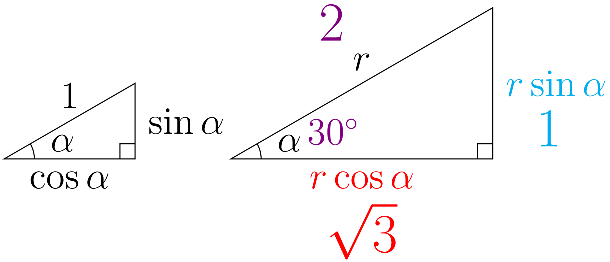
**問 2**　(1)　$\sqrt{3}\sin\theta + \cos\theta$

$$\underset{\displaystyle r\cos\alpha}{\underset{\downarrow}{\sqrt{3}}}\sin\theta + \underset{\displaystyle r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$



**問 2**　(1)　$\sqrt{3}\sin\theta + \cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{\sqrt{3}}}\sin\theta + \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$



**問 2** (1) $\sqrt{3}\sin\theta+\cos\theta$

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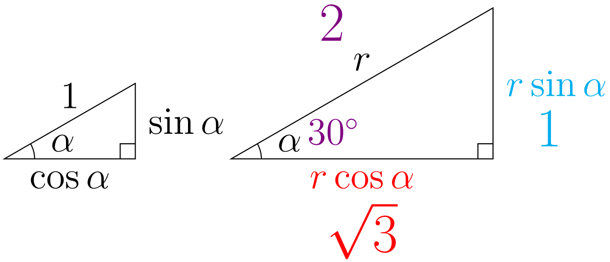


$$\sqrt{3}\sin\theta+\cos\theta=2\cos 30^\circ\sin\theta+2\sin 30^\circ\cos\theta$$

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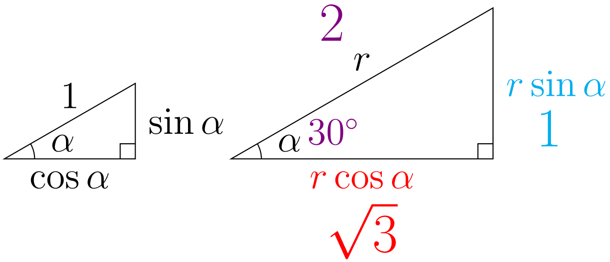
**問 2**　(1)　$\sqrt{3}\sin\theta + \cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{\sqrt{3}}}\sin\theta + \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$




$$\sqrt{3}\sin\theta + \cos\theta = 2\cos 30^\circ \sin\theta + 2\sin 30^\circ \cos\theta$$

$$= 2\sin(\theta + 30^\circ)$$

**問 2**　(2)　$\sin\theta - \cos\theta$

**問 2**　(2)　$\sin\theta - \cos\theta$

$$\underset{\displaystyle r\cos\alpha}{\underset{\downarrow}{1}}\sin\theta - \underset{\displaystyle r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$

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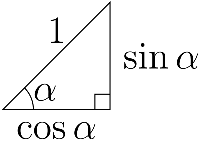
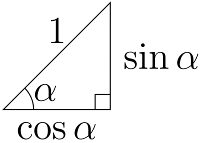
**問 2**　(2)　$\sin\theta - \cos\theta$

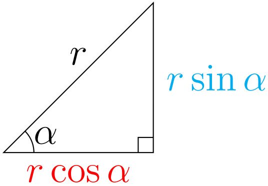
$$\underset{r\cos\alpha}{\underset{\downarrow}{1}}\sin\theta - \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$



**問 2**　(2)　$\sin\theta - \cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{1}}\sin\theta - \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$



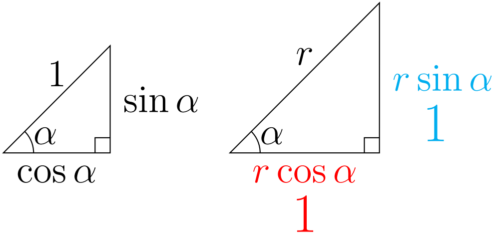



**問 2** (2) $\sin\theta - \cos\theta$

$$1\sin\theta - 1\cos\theta$$

$$\downarrow \qquad \downarrow$$

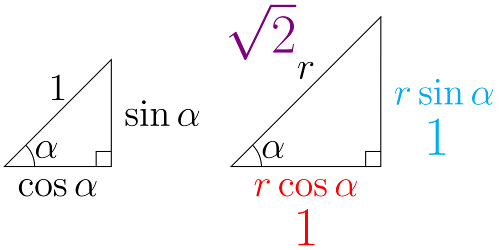
$$r\cos\alpha \qquad r\sin\alpha$$



**問 2**　(2)　$\sin\theta - \cos\theta$

$$1\sin\theta - 1\cos\theta$$

$$1 \downarrow r\cos\alpha \qquad 1 \downarrow r\sin\alpha$$

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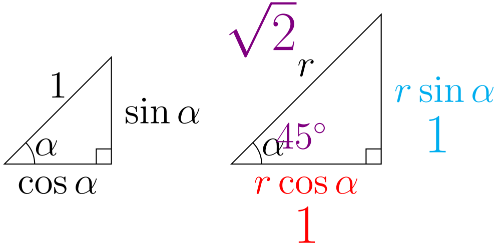
**問 2**　(2)　$\sin\theta - \cos\theta$

$$1\sin\theta - 1\cos\theta$$

$$\downarrow \qquad\qquad \downarrow$$

$$r\cos\alpha \qquad r\sin\alpha$$



## 問 2　(2)　$\sin\theta - \cos\theta$

$$1\sin\theta - 1\cos\theta$$

$$\downarrow \qquad \downarrow$$

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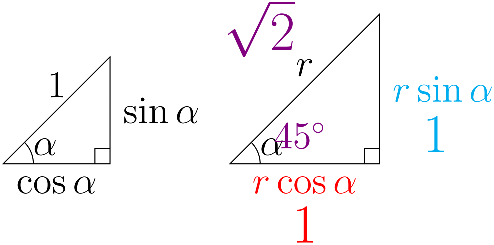
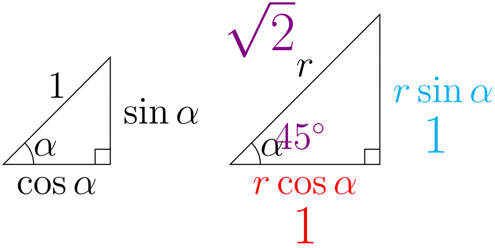



$$\sin\theta - \cos\theta = \sqrt{2}\cos 45^\circ \sin\theta - \sqrt{2}\sin 45^\circ \cos\theta$$

**問 2**　(2)　$\sin\theta - \cos\theta$

$$\underset{r\cos\alpha}{\underset{\downarrow}{1}}\sin\theta - \underset{r\sin\alpha}{\underset{\downarrow}{1}}\cos\theta$$




$$\sin\theta - \cos\theta = \sqrt{2}\cos 45^\circ \sin\theta - \sqrt{2}\sin 45^\circ \cos\theta$$

$$= \sqrt{2}\sin(\theta - 45^\circ)$$

# 今回の学習目標

三角関数の合成

- 図を用いて解を導く

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